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Vehicle-Bridge-Interaction Analysis Using Half-Car Model

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Abstract

In this study, the dynamic interaction that occurs between a vehicle and a flexible structure such as a bridge, is discussed in terms of random irregularities on the road of the bridge. The simple supported bridge used in the study is modeled using the Euler-Bernoulli beam theory. The vehicle body and wheels are taken into account as half vehicle model consisting of four degrees of freedom. Motion equation of the vehicle was obtained using the Lagrange method that evaluates the energy equations of the generalized coordinates. Then these equations were combined with equations representing the bridge system to obtain differential equations of the vehicle bridge interaction. This differential equations were solved using the Newmark β integration method with a special software in MATLAB

Key words: Vehicle-Bridge-Interaction, Half-Car Model, Random irregularities

1. Introduction

The dynamic behaviour of structure under the influence of moving loads as an important issue in engineering has taken place in the literature. For analytical solution of various problems of moving loads, the studies[1], [2] are essential in this field. Considering dynamic behaviour of structures under moving loads with variable velocity,[3]–[5] have studied the dynamic behaviour under the influence of accelerating mass on beams of different types. On a flexible structure without neglecting the effects of damping and inertia of the mass is a moving mass problem and some accurate solutions of these kind of problems using Finite element method (FEM) can be found in [6]–[9]. Moving mass problems in defence systems, for the interaction projectile and barrels are applicable and studies [10]–[13] are significant for the forced vibrations of the barrels due to the projectile and barrel interaction. One of the main applications of the vehicle bridge interaction problems that are the studies in this field are generally divided into nine categories. The effect of the suspension systems, road surface roughness effects, bridge length, the vehicle braking, the vehicle mass, vehicle speed, bridge damping, bridge unit length of the mass, the effect of the acceleration of the vehicle [14]. Despite The early simple cases of a moving concentrated force on a simply supported beam with constant speed[1], the vehicle bridge interaction particularly in bridge engineering applications, are quite important. The first studies in the literature in this area is focused the dynamics of bridges but vehicle dynamics is neglected[15], [16]. Later moving mass with constant or variable velocity over a railway bridge is an important research topic today and some useful solutions of so simple cases of it can be found in [17]. For evaluating the vehicle, bridge interaction in terms of passenger comfort of a vehicle on a flexible structure for constant velocity has been given in [18]. One of the major applications of train / rail interaction, for example, [19] proposed a two-axle vehicle on an Euler-Bernoulli beam the bridge interaction using finite element model. The other Wheel/rail interaction

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applications using FEM can be found in[20]–[22] . Another research interest in this field is on the effect of road roughness.
In general, depending of the accuracy of the modelling of both vehicle and bridge models, the vehicle-bridge interaction problems are very complicated in order to analysing the interaction considering the passenger comfort. In this study, the effects of basic parameters such as vehicle velocity, body mass and tire stiffness that affect the system are studied using a half car model.

2. Mathematical Modelling

Vehicle bridge interaction (VBI) is related to the coupled interaction between the vehicle and bridge. While the vehicle move on the bridge span, depending on the velocity of the vehicle the bridge is forced to vibrate, but the vibration of the bridge is coupled with the vehicle. This is called coupled vibration interaction between the vehicle and bridge. Depending on the mass, velocity, acceleration or decelerations, etc., the interaction should be analysed in terms of both vehicle stability on bridges and damage prevention of the bridges. Despite the coupling of the equation of motions of whole system, using a time step integration solution algorithm of motion equations, the coupled equation can be solved using the interaction forces at the contact point of the tires for the two-sub system. If the vehicle is moving with a constant velocity on the bridge, in this case there will be no interaction in the horizontal direction, so the contact forces between the bridge and the vehicle will be only the vertical direction and this assumption is adopted in this study.

2.1. Modelling of the vehicle

Structural forms of the vehicles may be quite complicated depending on the type of the vehicles. This study focused on a half-car model that has four degrees of freedom

Figure 1 shows an Euler-Bernoulli beam and a four DOF half car model of the vehicle that moves from the left end to the right end on the beam with constant velocity v. Where y_s and θ are respectively the vertical displacements of the vehicle body and the pitching of vehicle body, while y_{t1} and y_{t2} are respectively the vertical displacement of front and rear tires. On the other hand, parameters y_{c1} and y_{c2} are respectively at the contact points of the vehicle tires.

Before obtaining the differential equation of motion of the vehicle, the following basic assumptions are considered:

- Vehicle is moving with a constant velocity on the bridge.
- Wheel of the vehicle is always in contact with the bridge.
- Small displacements are in the frame of the Hookes's Law.
- Road roughness is not included in the models due to the page limitation of this study.

Considering Figure 1, for the half car model, the kinetic, potential energy, and Rayleigh's dissipation function are as follows, respectively:

$$E_k = \frac{1}{2}\left\{ m_s \dot{y}_s^2 + J\dot{\theta}^2 + m_{t1}\dot{y}_{t1}^2 + m_{t2}\dot{y}_{t2}^2 \right\} \tag{1a}$$

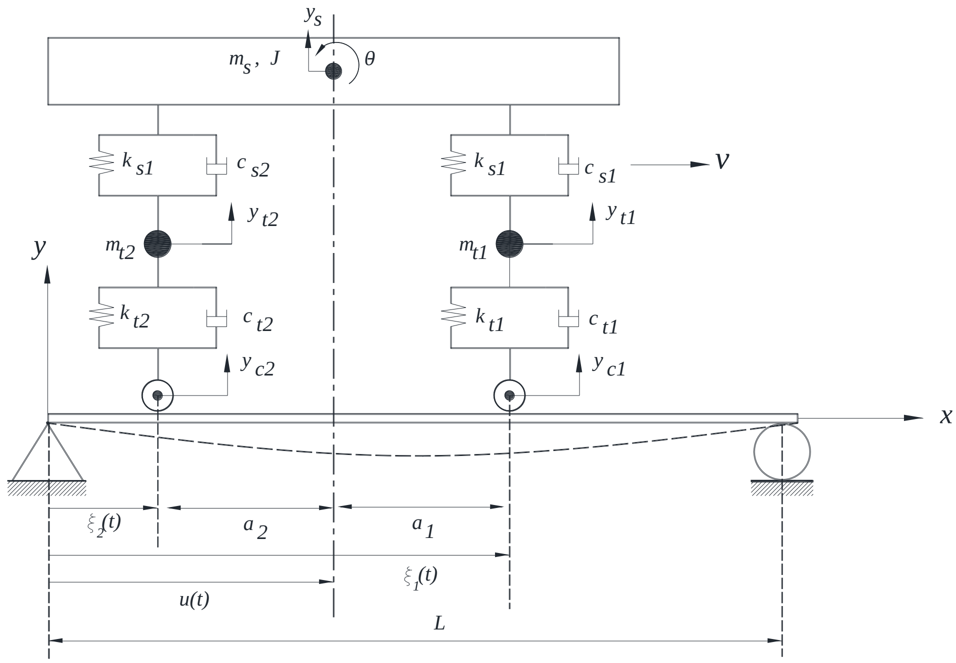

Figure 1. A four DOF half car model and an Euler-Bernoulli bridge beam.

$$E_p = \frac{1}{2}\left\{k_{s1}(\mathrm{y}_s + \theta a_1 - y_{t1})^2 + k_{s2}(\mathrm{y}_s - \theta a_1 - y_{t2})^2 + k_{t1}(\mathrm{y}_{t1} - y_{c1})^2 + k_2(\mathrm{y}_{t2} - y_{c2})^2\right\} \tag{1b}$$

$$D = \frac{1}{2}\left\{c_{s1}(\dot{\mathrm{y}}_s + \dot{\theta} a_1 - \dot{\mathrm{y}}_{t1})^2 + c_{s2}(\dot{\mathrm{y}}_s - \dot{\theta} a_2 - \dot{\mathrm{y}}_{t2})^2 + c_{t1}(\dot{\mathrm{y}}_{t1} - \dot{\mathrm{y}}_{c1})^2 + c_{t2}(\dot{\mathrm{y}}_{t2} - \dot{\mathrm{y}}_{c2})^2\right\} \tag{1c}$$

Lagrangian of the system as the difference between the potential energy and kinetic energy L is written as follows:

$$\frac{d}{dx}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = Q_i \quad (i = 1, 2, ..., n) \tag{2}$$

According to generalized coordinates of the system and Lagrange equation, the equations of motion of a half-car model is obtained as follows:

$$\{F_v\} = [M_v]\{\ddot{y}_v\} + [C_v]\{\dot{y}_v\} + [K_v]\{y_v\} \tag{3}$$

In Eq.(3), $[M_v]$, $[C_v]$ and $[K_v]$ represent mass, damping and stiffness matrix of entire system respectively. On the other hand, the parameters $\{F_v\}$ and $\{y_v\}$ represent force and displacement vector of whole system.

2.2. Modelling of the bridge

For the system shown in Figure 2 according to the Euler-Bernoulli beam theory equations of motion it can be expressed as follows:

$$EI\frac{\partial^4 w_b(\mathrm{x,t})}{\partial x^4}+\rho\frac{\partial^2 w_b(\mathrm{x,t})}{\partial t^2}+\mu\frac{\partial w_b(\mathrm{x,t})}{\partial t}=-F(\mathrm{x,t})\delta(\mathrm{x}-\mathrm{vt}) \tag{4}$$

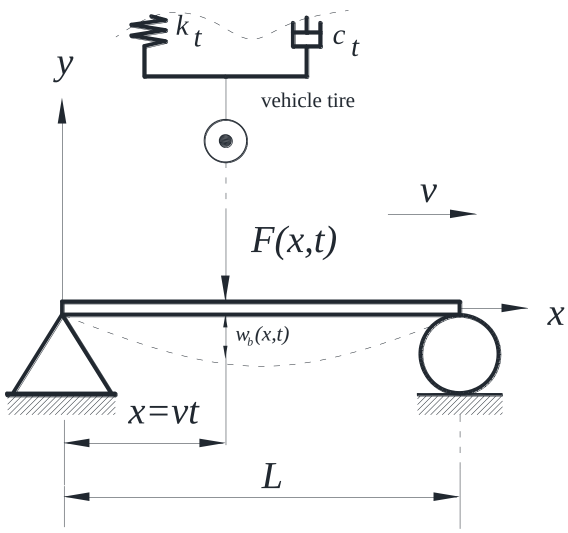

Figure 2. An Euler-Bernoulli bridge beam with a moving force.

In Eq. (4), the expression E represents the modulus of elasticity of the bridge girder, I represents bridge area moment of inertia of beam cross-section, ρ represents mass per unit length of the beam, μ represents the equivalent viscous damping coefficient for bridge beam girders, and δ represents the Dirac delta function. The time dependent displacements of the beam at x and time t can be approximated as follows:

$$\mathrm{w}_b(\mathrm{x,t})=\sum_{k=1}^{N}\phi_k(\mathrm{x})\,\mathrm{q}_k(\mathrm{t}), \tag{5}$$

Where $\phi_k(x)$ and $q_k(t)$, (k=1,2,..N) are, respectively, represents mode shapes and modal coordinates. Vibration mode shapes of a simply supported beam bridge can be expressed as follows:

$$\phi_k(\mathrm{x})=\sqrt{\frac{2}{\rho L}}\sin\frac{k\pi x}{L}, \tag{6}$$

$$\ddot{q}_n+2\lambda_n\omega_n\dot{q}_n+\omega_n^2 q_n=-F(\mathrm{x}=\mathrm{vt,t})\phi_n(\mathrm{x}=\mathrm{vt}) \tag{8}$$

$$\lambda_n = \frac{\mu}{2\rho\omega_n}, \quad \phi_n(vt) = \sqrt{\frac{2}{\rho L}} \sin\frac{n\pi vt}{L}, \tag{9}$$

2.3. Coupling the motion equations

When wheel moves on the bridge, there are two type of displacements: dynamic deformation of the beam and surface roughness at the contact point of the bridge. In this case, the displacement and velocity of the wheel contact point is obtained as follows:

$$y_{c,i} = w_{b,i}(\mathrm{x,t}) + \mathrm{r}_i(\mathrm{x}) = \sum_{k=1}^{N} \phi_{k,i}(\mathrm{x})\,\mathrm{q}_{k,i}(\mathrm{t}) + r_i, \quad i = 1,2 \tag{10a}$$

$$\dot{y}_{c,i} = \sum_{k=1}^{N} \dot{\phi}_{k,i}(\mathrm{x})\,\mathrm{q}_{k,i}(\mathrm{t}) + \sum_{k=1}^{N} \phi_{k,i}(\mathrm{x})\,\dot{\mathrm{q}}_{k,i}(\mathrm{t}) + \dot{r}_i, \quad i = 1,2 \tag{10b}$$

The forces acting on the bridge from wheel are in two type that one is static load of the vehicle and the other is the dynamic load of the vehicle. Considering these, the forces acting on bridge from the vehicle at the bridge contact point can be expressed as follows:

$$F_i(\mathrm{x,t}) = \mathrm{W}_i - \mathrm{k}_{t,i}(\mathrm{y}_{t,i} - \mathrm{y}_{c,i}) - \mathrm{c}_{t,i}(\dot{\mathrm{y}}_{t,i} - \dot{\mathrm{y}}_{c,i}), \quad \mathrm{i=1,2} \tag{11a}$$

$$F_i(x,t) = W_i + m_{t,i}\ddot{y}_{t,i} + m_{s,i}\ddot{y}_{s,i}, \;\; i = 1,2. \tag{11b}$$

Imposing the contact forces from Eq. (11a) and (11b) in Eq. (8), the following motion equations are obtained:

$$q_n + 2\lambda_n\omega_n\dot{q}_n + \omega_n^2 q_n + \phi_n\delta m_t\ddot{y}_t + \phi_n\delta m_s\ddot{y}_s = -W\phi_n\delta, \tag{12}$$

$$M(\mathrm{t})\,\ddot{\mathrm{Y}} + \mathrm{C(t)}\,\dot{\mathrm{Y}} + \mathrm{K(t)}\,\mathrm{Y} = \mathrm{F}_Q(\mathrm{t}), \tag{13}$$

3. Results

In this section, vehicle bridge-induced vibration and influences on the vibration of vehicle will be examined. The vehicle wheel and vehicle body vibration will be examined in terms of acceleration and displacements. Figures (3a and 3b) show the displacement and acceleration of the vehicle body at different vehicle velocities, respectively. As it is shown from Figure (3a), vehicle body displacements changes but the change is not linear depending on the velocity increase of the vehicle. Figure 3b shows the accelerations of the vehicle body for different velocities of the vehicle. The increase of the vehicle velocity, as shown in the Figure 3b, it shows a linear relationship between the acceleration of the body. For example, the accelerations of the vehicle body for 40, 80, 120 and 160 km/h, the maximum values of the accelerations, respectively, are 0.011, 0.0307, 0.059 and 0.076 m/s^2. In addition, it is seen that the maximums occur when the vehicle is on about 75% of the bridge length.

Bridge and vehicle parameters used in this study are presented in Table 1. In Figure 4, the displacements of the bridge midpoint for different velocity of vehicle are given. Although it is not a linear relationship between the vehicle speeds, maximum displacement of bridge mid-point increases with the increase of the vehicle speed, in general. In addition, the maximum midpoint displacement of the bridge does not occur when the vehicle is on the middle of the bridge. For lover velocities, it can be before mid-point, and for higher velocities, it is generally after the passage of the mid-point. The maximum bending moment and displacements occur around of the midpoint of the bridge by + - 20% location change [18].

The dynamic response of the vehicle is an important issue in terms of both driving safety and passenger comfort. While the vehicle travels over the bridge beam, the motif of the bridge affects the motion of the vehicle and vice-versa. Structural engineers are generally interested in the dynamics of the bridge beams for predicting impact factors of vehicle motions. However, in terms of transportation safety and passenger comfort the problem of the vehicle bridge interaction has different aspects of interests. Especially the displacements and accelerations of the vehicle components are very important in order to design of these components, which experience time dependent dynamic forces. Moreover, for a smooth drive of the vehicle on the bridge, the contact of the wheels and the road surface should always be satisfied in order to prevent the vehicle from unstable driving conditions. For passenger comforts, the acceleration and displacements of the vehicle may adversely affects the physiological and psychological health of both drivers and passengers.

Figures (5a) shows the displacements of the vehicle body for a constant velocity of the vehicle and different body masses of the vehicle, while Figure 5b is showing the acceleration of the vehicle body. For example for a velocity of 90 km/h, both the accelerations and displacements are increased with the increasing of the body mass

Figure 6 shows the effect of the vehicle body mass m_s on the displacements of bridge mid-point. From Figure 6, one can realize that any increase of the vehicle body mass, increases the dynamic displacements. However, unlike the effect of the velocity, the effects of the mass on the midpoint displacements are generally linear. The positions of the maximums do not change severely as it was for velocity. The maximums occur after the mid- point and they move slightly to the right end with increasing velocity. From these results, it is obvious that the vibration characteristics of the bridge are severely affected form vehicle velocity when compared the effect of the vehicle mass.

Table 1. The numerical values of the vehicle and bridge used in this study.

Bridge		Vehicle parameters			
L (m)	50	m_s (kg)	1794.4	k_{t2} (N/m)	101115
E (Gpa)	207	m_{t1} (kg)	87.15	c_{s1} (Ns/m)	1190
I (m^4)	0.174	m_{t2} (kg)	140.4	c_{s2} (Ns/m)	1000
μ (kg/m)	20000	J (kgm^2)	3443.04	c_{t1} (Ns/m)	14.6
c (Ns/m)	1750	a_1 (m)	1.271	c_{t2} (Ns/m)	14.6
		k_{s1} (N/m)	66824.2	a_2 (m)	1.713
		k_{s2} (N/m)	18615	k_{t1} (N/m)	101115

Figure 7 shows the effects of the tire stiffness on the dynamics of the vehicle for a travelling speed of v=90 km/h, and integration time step size Δt=0.0001 s. Accepting that the both tire stiffness are equal $k_{t1}=k_{t2}$, and starting from 10^5 to 4.10^5 N/m with a increment of $2x10^4$. As seen from Figure the body accelerations are reduced with the increasing stiffness of the tires. However, the accelerations of the tires are increased. The pithing acceleration ($\ddot{\theta}$) of the body has reached the maximum at $k_{t1}=k_{t2}=2x10^5$, but after this value it has been decreased by the increasing of the tire stiffness.

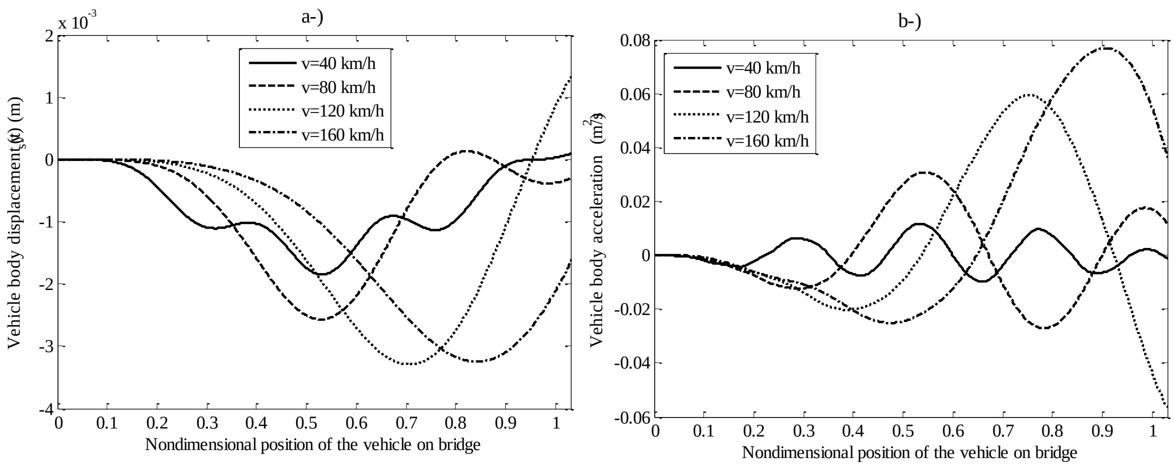

Figure 3. Dynamic analysis of vehicle for time step size Δt=0.0001 and four vehicle constant velocities; a-) vehicle body displacement (m); b-) vehicle body acceleration (m/s^2)

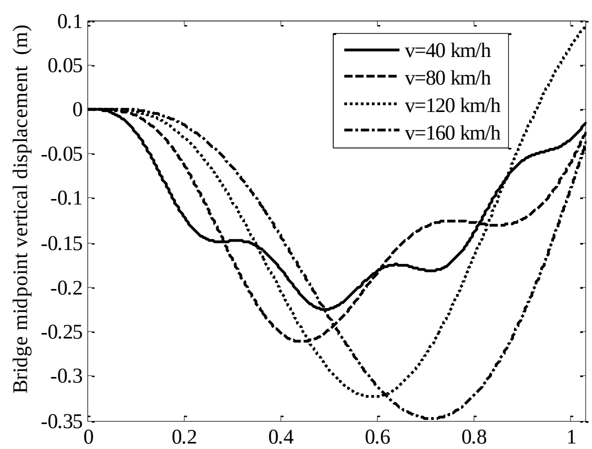

Figure 4. Dynamic analysis of bridge mid-point for time step size Δt=0.0001 and four vehicle constant velocities;

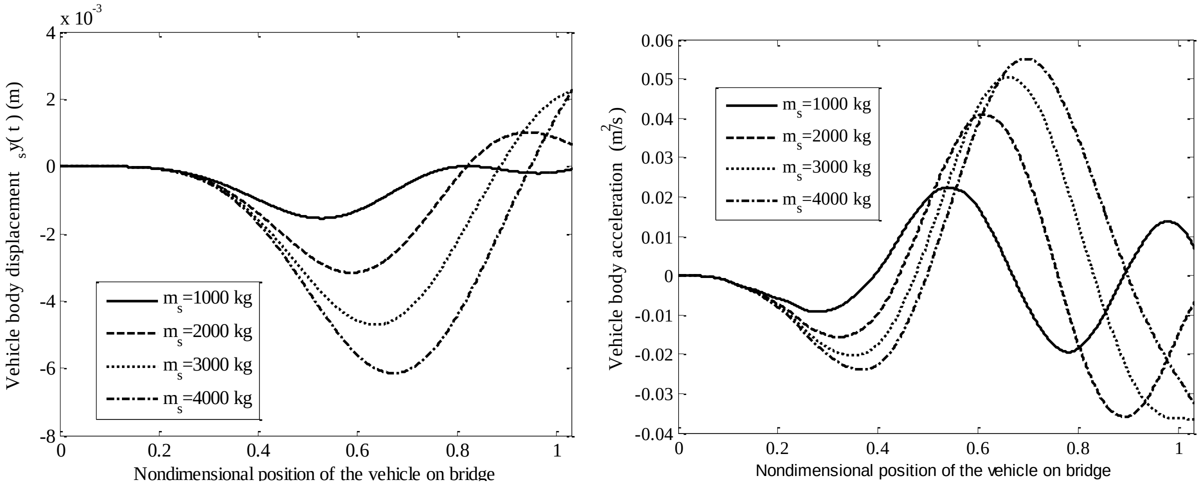

Figure 5. Dynamic analysis of vehicle for time step size Δt=0.0001 and four vehicle body masses; a-) vehicle body displacement (m); b-) vehicle body acceleration (m/s^2).

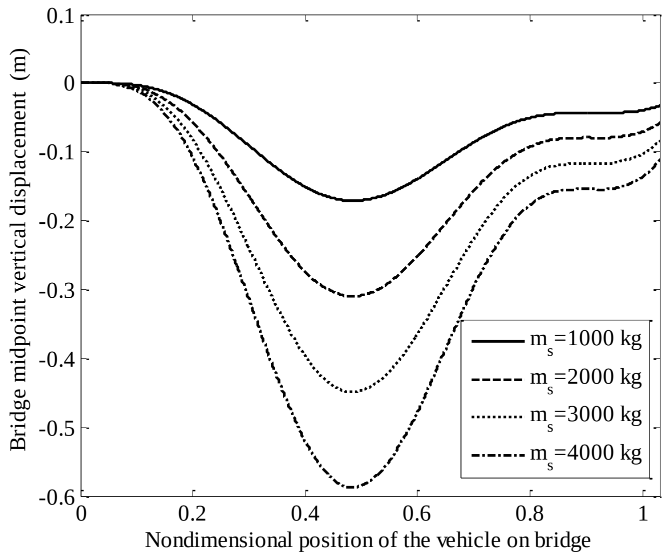

Figure 6. Dynamic analysis of bridge mid-point for time step size Δt=0.0001 and four vehicle body masses.

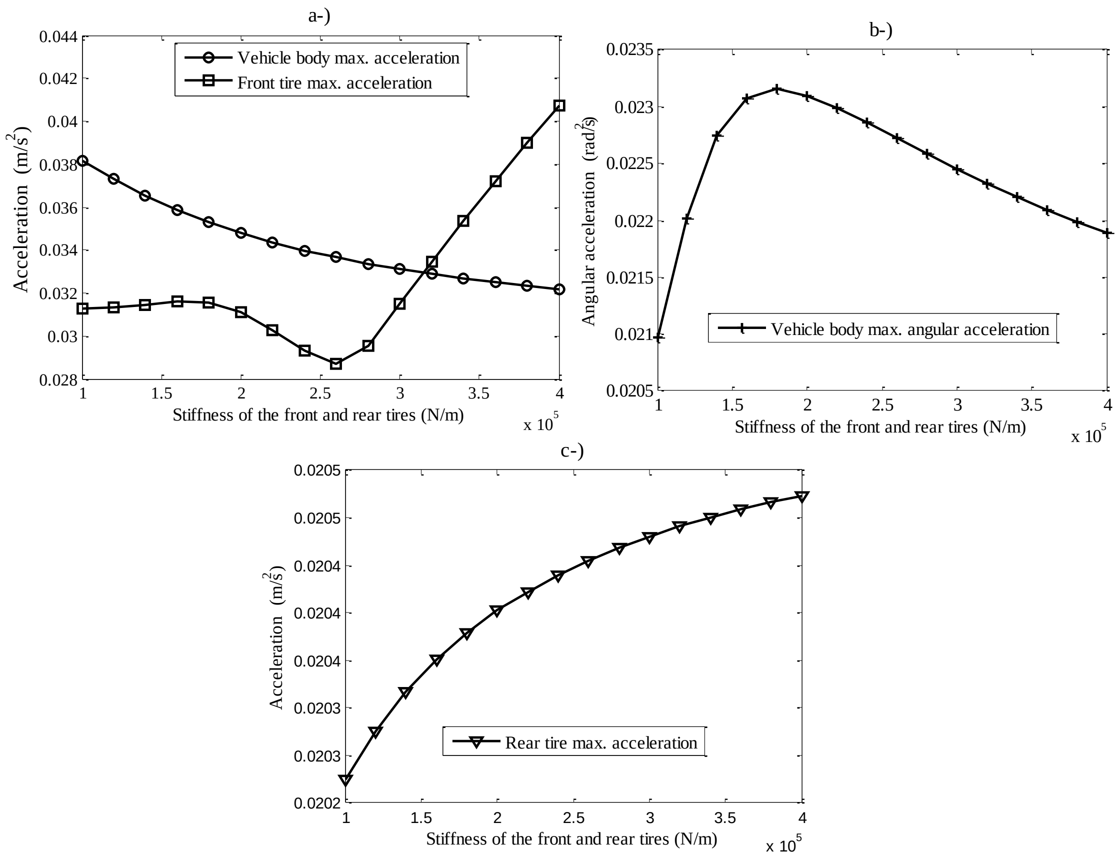

Figure 7. The effect of front and rear tire stiffness (k_{t1} and k_{t2}) upon vehicle dynamics; a-) vehicle body and front tire acceleration; b-) vehicle body angular acceleration; c-) rear tire acceleration (rad/s^2).

4. Discussion

In this study, the interaction of a vehicle with a bridge is studied and the interaction is investigated both the dynamics of vehicle and bridge. The equations of motions of vehicle and bridge sub systems are coupled. These equations are uncoupled using modal shapes of the vibration modes and time dependent modal coordinates; and taking the advantages of orthogonality of individual mode shapes. Finally using a precise time step integration, the equations of the motions of whole system are solved.

In general, many parameters affect the interaction of a vehicle and bridge such as vehicle speed, vehicle mass, vehicle axles, bridge length, bridge rigidity, road roughness etc. This study is focused on both the vehicle and bridge dynamics depending on mass and the velocity of the vehicle, and the following results are obtained:

- In general, the increase of vehicle travelling speed is increasing displacement of the midpoint of the bridge. However, in some low speeds this generality is not valid.
- The maximum displacement does not occur at mid-point but around the midpoint with a 20% of the length of the bridge from the midpoint. If the vehicle velocity is increased, it also increases the wavelength of the oscillation of the bridge, but decreases the amplitude

of the displacements for both vehicle and bridge. For both vehicle and bridge, the accelerations always are increased by any increase in vehicle velocity.

- The increase in mass of the vehicle increased as well as the accelerations and displacements of the vehicle body However, the maximum displacement and acceleration does not change location on the bridge.
- Vehicle body vertical acceleration is reduced by increasing of the tires stiffness. But increasing of vehicle tire stiffness causes to occur bigger front and rear tire acceleration.

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